Solutions for the Ill-posed Problem of Inverse Calculating Three-phase Voltages of Overhead Transmission Lines by Using Power-frequency Electric Field Data

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A new idea is proposed to realize the non-contacted voltage measurement of overhead AC transmission lines, which is to inversely calculate the voltage parameters by using the power-frequency electric field measurement data. However, the inverse calculation is serious ill-posed problem. Two-step solution is proposed to deal with the ill-posed problem. Firstly, the optimal positions of the measuring points are searched based on the particle swarm optimization algorithm. It can significantly reduce the condition number of observation matrix. Secondly, iterated Tikhonov regularization is used to further improve the calculation. The simulation results show that the proposed method can improve the accuracy and stability of the voltage parameters' identification even at a large noise level.

***Index Terms*—overhead AC transmission line; electric field; voltage; ill-posed problem; position optimization; iterated Tikhonov regularization**

I. Introduction

ADDING new potential transformer in existing overhead transmission lines (OTLs) faces many problems. Thus, new ways of monitoring the voltage of OTLs with safe and reliable performance need to be developed.

Numerous studies on the electromagnetic environment of high-voltage AC OTLs reveal that the power-frequency electric field is significantly correlated with the voltages on OTLs [1]. Thus, we propose the innovative idea to inversely calculate the voltage characteristic parameters, i.e. amplitude and phase, by using the power-frequency electric field data measured under OTLs. In this way the non-contacted voltage measurement of AC OTLs can be realized.

However, the inverse calculation is a serious ill-posed problem. Errors inevitably exist in actual electric field measurement, and then the inverse results may severely deviate from the true values. In order to deal with the problem, two-step solution is proposed.

II. Mathematical Model and Optimization method

A. Mathematical Relationship between Power-frequency Voltage and Electric Field of Overhead Transmission Lines

The analog line charge is set in the equivalent conductor based on the Charge Simulation Method [1]. The relationship between the voltage $\boldsymbol{U}$ and the analog charge $\boldsymbol{\tau}$ is

$$\boldsymbol{U} = \boldsymbol{P}\boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{P}$ is the potential coefficient matrix and its elements can be obtained by the principle of mirror image.

The electric field of the measuring points $o(x_o, y_o)$ can be calulated with $\boldsymbol{\tau}$

$$\boldsymbol{E} = \boldsymbol{G}\boldsymbol{\tau} \tag{2}$$

The elements in matrix $\boldsymbol{P}$ and $\boldsymbol{G}$ are determined by the positions of the measuring points and the phase conductors.

The mathematical relationship between the voltages and electric field of OTLs can be expressed as

$$\boldsymbol{E} = \boldsymbol{K}\boldsymbol{U} \tag{3}$$

where $\boldsymbol{K}=\boldsymbol{G}\boldsymbol{P}^{-1}$ is defined as the observation matrix.

If the positions of the measuring points are selected randomly, the condition number of the matrix $\boldsymbol{K}$ may be much large, which means the inverse calculation is a serious ill-posed problem. Consequently, the small noise in $\boldsymbol{E}$ may cause the inverse solution to severely deviate from the true solution.

B. Position Optimization of Measurement Points Based on Improved Particle Swarm Optimization Algorithm

The condition number of the observation matrix $\boldsymbol{K}$ is the main index to reflect the ill-posedness of the inverse problem. The matrix $\boldsymbol{K}$ is calculated according to the positions of electric field measuring points. Therefore, reasonable selection of the measuring points' positions can reduce the condition number of matrix $\boldsymbol{K}$. Here Particle Swarm Optimization Algorithm is adopted.

The fitness function is the condition number of matrix $\boldsymbol{K}$.

$$\mathrm{FitFun} = \mathrm{cond}(\boldsymbol{K}) \tag{4}$$

The variables are the coordinate positions of N measuring point in the x and y axes, i.e. $\boldsymbol{X}_m = [x_{m1}, \cdots, x_{mn}, \cdots, x_{mN}]$ and $\boldsymbol{Y}_m = [y_{m1}, \cdots, y_{mn}, \cdots, y_{mN}]$, where M is the number of particles.

Set the number of mesuring points and the specific constraints, and successfully search for the global optimal solution. The iteration formula for the velocity and position of the particle swarm in the (t+1)th generation are as follows [2]:

$$\boldsymbol{V}_{x,m}^{t+1} = \omega \boldsymbol{V}_{x,m}^{t} + c_1 r_{x1}^{t+1}(X_{Hbest,m} - \boldsymbol{X}_m^t) + c_2 r_{x2}^{t+1}(X_{Gbest} - \boldsymbol{X}_m^t) \tag{5a}$$

$$\boldsymbol{V}_{y,m}^{t+1} = \omega \boldsymbol{V}_{y,m}^{t} + c_1 r_{y1}^{t+1}(Y_{Hbest,m} - \boldsymbol{Y}_m^t) + c_2 r_{y2}^{t+1}(Y_{Gbest} - \boldsymbol{Y}_m^t) \tag{5b}$$

$$\boldsymbol{X}_m^{t+1} = \boldsymbol{X}_m^t + \boldsymbol{V}_{x,m}^{t+1} \tag{6a}$$

$$\boldsymbol{Y}_m^{t+1} = \boldsymbol{Y}_m^t + \boldsymbol{V}_{y,m}^{t+1} \tag{6b}$$

where ω is the inertia weight, c_1 and c_2 are learning factors, r_{x1}^{t+1}, r_{x2}^{t+1}, r_{y1}^{t+1} and r_{y2}^{t+1} are random numbers that value from 0 to 1 respectively.

The iterative termination condition is reaching the maximum number of iterations or the preset fitness threshold value.

C. Inverse Calculation Based on Iterated Tikhonov Regularization

If search space range for the optimal position is limited to be adjacent ground, the corresponding cond($\boldsymbol{K}$) is still not particularly satisfied, but it is much less than the result obtained by selected randomly selecting the measuring points. In view of this situation, it is necessary to do further processing.

The (3) is transformed into the minimization problem

$$\min \boldsymbol{J}_\alpha(\boldsymbol{U}), \quad \boldsymbol{J}_\alpha(\boldsymbol{U}) = \| \boldsymbol{KU} - \boldsymbol{E}^\delta \|^2 + \alpha \| \boldsymbol{U} \|^2 \qquad (7)$$

where α is the regularization parameter.

Iterated Tikhonov regularization is adopted to improve the inverse calculation [3].

$$\begin{cases} \boldsymbol{U}_\alpha^{0,\delta} = 0 \\ \boldsymbol{U}_\alpha^{m,\delta} = \alpha(\boldsymbol{K}^*\boldsymbol{K} + \alpha \boldsymbol{I})^{-1}\boldsymbol{U}_\alpha^{m-1,\delta} + (\boldsymbol{K}^*\boldsymbol{K} + \alpha\boldsymbol{I})^{-1}\boldsymbol{K}^*\boldsymbol{E}^\delta \end{cases} \qquad (8)$$

where $\boldsymbol{K}$* is the adjoint operator of $\boldsymbol{K}$.

The crucial points of the iterated regularization are to set α and termination condition of iterative procedure.

III. Simulation Details and Results

Fig.1 shows the layout of the three-phase conductors in the 220 kV overhead transmission lines system. The analytical conditions are:

1) The three-phase voltages of OTLs are symmetrical.

$$\boldsymbol{U}^T = [127.02\angle 0^\circ \quad 127.02\angle -120^\circ \quad 127.02\angle 120^\circ]\text{kV}$$

2) Only three measuring points are set and they are symmetrically placed.

3) The signal-to-noise ratio of 15 dB is set and a random Gauss white noise is added.

If the measuring points are randomly set (-5.5 1.5), (0 1.5), (5.5 1.5) without the position optimization. Then, cond($\boldsymbol{K}$)=98.90, and the voltage inverse solutions are

$$\boldsymbol{U}^{\delta\,T} = [199.50\angle -5.8^\circ \quad 420.97\angle -71.2^\circ \quad 114.77\angle 40.7^\circ]\text{kV}.$$

Obviously, this result is undesirable.

In view of the limitations of the actual measurement conditions, the position of the measuring points can only be selected within a small space. Basing on our previous study, we propose two measurement schemes: making a measurement near the ground and making a measurement near the conductors.

In the first measurement scheme, the search scope is set as $-15\ \text{m} \le x_k \le 15\ \text{m}$ and $1\ \text{m} \le y_k \le 3\ \text{m}$. The global optimal solutions of ten times of optimization process are same and they are (-15 3), (0 1), (15 3). Then, cond($\boldsymbol{K}$)=23.56, and the voltage inverse solutions are

$$\boldsymbol{U}^{\delta\,T} = [141.52\angle -3.2^\circ \quad 138.25\angle -137.6^\circ \quad 118.34\angle 125.7^\circ]\text{kV}.$$

In the second measurement scheme, the search scope is set as $-10\ \text{m} \le x_k \le 10\ \text{m}$ and $12\ \text{m} \le y_k \le 14\ \text{m}$. The global optimal solutions of ten times of optimization process are also same and they are (-9 12), (0 14), (9 12). Then, cond($\boldsymbol{K}$)=1.46, and the voltage inverse solutions are

$$\boldsymbol{U}^{\delta\,T} = [126.88\angle 2.2^\circ \quad 123.17\angle -117.2^\circ \quad 132.75\angle 117.4^\circ]\text{kV}.$$

It shows that the results of the proposed position optimization can improve the ill-posed problem of the inverse calculation. The inverse solutions obtained from the measurements near the conductors are superior to those obtained from the measurements near the ground. However, measuring near the ground has the advantages of simple operation, flexibility and safety. In order to improve the calculation accuracy in this condition, iterative Tikhonov regularization is used. The optimal solution can be obtained after 4 iterations under setting α=6.5*10^{-6} and $\| \boldsymbol{KU}_\alpha^{m,\delta} - \boldsymbol{E}^\alpha \| \le 1.1\delta$. Fig.2 shows the waveforms of three-phase voltages in time domain in each iterative calcualtion.

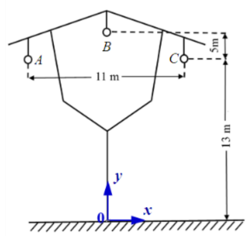

Fig.1. Space layout of 220 kV OTLs

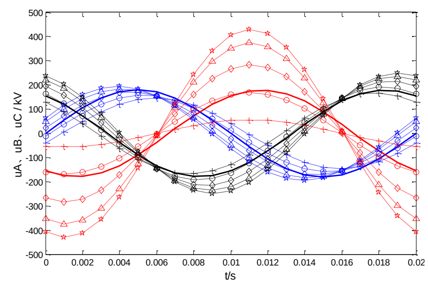

Fig.2. Waveforms of three-phase voltages in each iterative calcualtion

Comparing with the computational process using only regularization method, the choice for optimal α becomes more easier with the proposed two-step method. The accurcy and speed of iterative calculation are improved.

IV. Conclusion

The paper proposes a method to deal with the ill-posed problem of inverse calculating the three-phase voltages of OTLs from the measuring electric field data. It combines position optimization of measurement points and iterative regularization. The simulation examples show that the proposed method can significantly improves the accuracy and stability of the inverse solution.

References

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